

# Michigan Merit Curriculum

## Course/Credit Requirements



# PRECALCULUS

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1 Credit (optional)





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## Welcome

This guide was developed to assist teachers in successfully implementing the Michigan Merit Curriculum. The identified content expectations and guidelines provide a successful framework for designing curriculum, assessments and relevant learning experiences for students. Through the collaborative efforts of Governor Jennifer M. Granholm, the State Board of Education, and the State Legislature, these landmark state graduation requirements are being implemented to give Michigan students the knowledge and skills to succeed in the 21st Century and drive Michigan's economic success in the global economy. Working together, teachers can explore varied pathways to help students demonstrate proficiency in meeting the content expectations and guidelines.

## Curriculum Unit Design

One of the ultimate goals of teaching is for students to acquire transferable knowledge. To accomplish this, learning needs to result in a deep understanding of content and mastery level of skills. As educational designers, teachers must use both the art and the science of teaching. In planning coherent, rigorous instructional units of study, it is best to *begin with the end in mind*.

### ***Engaging and effective units include***

- appropriate content expectations
- students setting goals and monitoring own progress
- a focus on big ideas that have great transfer value
- focus and essential questions that stimulate inquiry and connections
- identified valid and relevant skills and processes
- purposeful real-world applications
- relevant and worthy learning experiences
- varied flexible instruction for diverse learners
- research-based instructional strategies
- explicit and systematic instruction
- adequate teacher modeling and guided practice
- substantial time to review or apply new knowledge
- opportunities for revision of work based on feedback
- student evaluation of the unit
- culminating celebrations

## Relevance

Instruction that is clearly relevant to today's rapidly changing world is at the forefront of unit design. Content knowledge cannot by itself lead all students to academic achievement. Classes and projects that spark student interest and provide a rationale for why the content is worth learning enable students to make connections between what they read and learn in school, their lives, and their futures. An engaging and effective curriculum provides opportunities for exploration and exposure to new ideas. Real-world learning experiences provide students with opportunities to transfer and apply knowledge in new, diverse situations.

## Student Assessment

The assessment process can be a powerful tool for learning when students are actively involved in the process. Both assessment *of* learning and assessment *for* learning are essential. Reliable formative and summative assessments provide teachers with information they need to make informed instructional decisions that are more responsive to students' needs. Engagement empowers students to take ownership of their learning and builds confidence over time.

### **Sound assessments:**

- align with learning goals
- vary in type and format
- use authentic performance tasks
- use criteria scoring tools such as rubrics or exemplars
- allow teachers and students to track growth over time
- validate the acquisition of transferable knowledge
- give insight into students' thinking processes
- cause students to use higher level thinking skills
- address guiding questions and identified skills and processes
- provide informative feedback for teachers and students
- ask students to reflect on their learning

## High School Content Expectation Codes

To allow for ease in referencing expectations, each mathematics expectation has been coded by strand, standard, and expectation. For example:

PI.2	{	<b>P:</b> Precalculus strand
		<b>PI:</b> Standard I of the Precalculus strand
		<b>PI.2:</b> 2nd expectation in Standard PI

## Organizational Structure

<b>STRAND 1</b> Quantitative Literacy and Logic (L)	<b>STRAND 2</b> Algebra and Functions (A)
<b>STANDARDS</b> (and number of core expectations in each standard)	
<b>L1: Reasoning About Numbers, Systems and Quantitative Situations</b> (9) <b>L2: Calculation, Algorithms, and Estimation</b> (9) <b>L3: Measurement and Precision</b> (5) <b>L4: Mathematical Reasoning, Logic, and Proof</b> (10)	<b>A1: Expressions, Equations, and Inequalities</b> (16) <b>A2: Function</b> (39) <b>A3: Mathematical Modeling</b> (3)
<i>Recommended Quantitative Literacy and Logic Expectations (3)</i>	<i>Recommended Algebra and Functions Expectations (5)</i>
<b>STRAND 3</b> Geometry and Trigonometry (G)	<b>STRAND 4</b> Statistics and Probability (S)
<b>STANDARDS</b> (and number of core expectations in each standard)	
<b>G1: Figures and Their Properties</b> (29) <b>G2: Relationships Between Figures</b> (10) <b>G3: Transformations of Figures in the Plane</b> (5)	<b>S1: Univariate Data—Examining Distributions</b> (9) <b>S2: Bivariate Data—Examining Relationships</b> (6) <b>S3: Samples, Surveys, and Experiments</b> (3) <b>S4: Probability Models and Probability Calculation</b> (4)
<i>Recommended Geometry and Trigonometry Expectations (3)</i>	<i>Recommended Statistics and Probability Expectations (6)</i>

## **Recommended Course Credit**

The standards in this section, while not required for all students, are strongly recommended. These standards describe material to be taken after completion of the required high school content expectations. The knowledge and skills described in these standards will prepare students for postsecondary training, including apprenticeships and introductory college courses.

## **Organization of this Document**

In the mathematics credit requirement documents, the expectations are organized by strand and standard. The organization in no way implies an instructional sequence. Curriculum personnel or teachers are encouraged to organize these topics and expectations in a manner that encourages connections between strands.

## Introduction to Precalculus

Calculus is a powerful, useful, and versatile branch of mathematics. While the core ideas of calculus (derivatives and integrals) are not hard to understand, calculus is a demanding subject because it requires a broad and thorough background of algebra and functions. Study of the topics, concepts, and procedures of precalculus is very strongly recommended for all college-bound students. These topics, concepts, and procedures are prerequisites for many college programs in science, engineering, medicine, and business.

## Precalculus Goal Statement

Precalculus is the preparation for calculus. The study of the topics, concepts, and procedures of precalculus deepens students' understanding of algebra and extends their ability to apply algebra concepts and procedures at higher conceptual levels, as a tool, and in the study of other subjects. The theory and applications of trigonometry and functions are developed in depth. New mathematical tools, such as vectors, matrices, and polar coordinates, are introduced, with an eye toward modeling and solving real-world problems.

### *Precalculus Standards Outline*

<b>STANDARD P1:</b>	<b>FUNCTIONS</b>
<b>STANDARD P2:</b>	<b>EXPONENTIAL AND LOGARITHMIC FUNCTIONS</b>
<b>STANDARD P3:</b>	<b>QUADRATIC FUNCTIONS</b>
<b>STANDARD P4:</b>	<b>POLYNOMIAL FUNCTIONS</b>
<b>STANDARD P5:</b>	<b>RATIONAL FUNCTIONS AND DIFFERENCE QUOTIENTS</b>
<b>STANDARD P6:</b>	<b>TRIGONOMETRIC FUNCTIONS</b>
<b>STANDARD P7:</b>	<b>VECTORS, MATRICES, AND SYSTEMS OF EQUATIONS</b>
<b>STANDARD P8:</b>	<b>SEQUENCES, SERIES, AND MATHEMATICAL INDUCTION</b>
<b>STANDARD P9:</b>	<b>POLAR COORDINATES, PARAMETERIZATIONS, AND CONIC SECTIONS</b>

## CONTENT EXPECTATIONS FOR PRECALCULUS

### PI Functions

- P1.1 Know and use a definition of a function to decide if a given relation is a function.
- P1.2 Perform algebraic operations (including compositions) on functions and apply transformations (translations, reflections, and rescalings).
- P1.3 Write an expression for the composition of one given function with another and find the domain, range, and graph of the composite function. Recognize components when a function is composed of two or more elementary functions.
- P1.4 Determine whether a function (given symbolically or graphically) has an inverse and express the inverse (symbolically, if the function is given symbolically, or graphically, if given graphically) if it exists. Know and interpret the function notation for inverses.
- P1.5 Determine whether two given functions are inverses, using composition.
- P1.6 Identify and describe discontinuities of a function (e.g., greatest integer function,  $1/x$ ) and how these relate to the graph.
- P1.7 Understand the concept of limit of a function as  $x$  approaches a number or infinity. Use the idea of limit to analyze a graph as it approaches an asymptote. Compute limits of simple functions (e.g., find the limit as  $x$  approaches 0 of  $f(x) = 1/x$ ) informally.
- P1.8 Explain how the rates of change of functions in different families (e.g., linear functions, exponential functions, etc.) differ, referring to graphical representations.



## **P2 Exponential and Logarithmic Functions**

- P2.1** Use the inverse relationship between exponential and logarithmic functions to solve equations and problems.
- P2.2** Graph logarithmic functions. Graph translations and reflections of these functions.
- P2.3** Compare the large-scale behavior of exponential and logarithmic functions with different bases and recognize that different growth rates are visible in the graphs of the functions
- P2.4** Solve exponential and logarithmic equations when possible, (e.g.  $5^x = 3^{(x+1)}$ ). For those that cannot be solved analytically, use graphical methods to find approximate solutions.
- P2.5** Explain how the parameters of an exponential or logarithmic model relate to the data set or situation being modeled. Find an exponential or logarithmic function to model a given data set or situation. Solve problems involving exponential growth and decay.

## **P3 Quadratic Functions**

- P3.1** Solve quadratic-type equations (e.g.  $e^{2x} - 4e^{x+4} = 0$ ) by substitution.
- P3.2** Apply quadratic functions and their graphs in the context of motion under gravity and simple optimization problems.
- P3.3** Explain how the parameters of an exponential or logarithmic model relate to the data set or situation being modeled. Find a quadratic function to model a given data set or situation.

## **P4 Polynomial Functions**

- P4.1** Given a polynomial function whose roots are known or can be calculated, find the intervals on which the function's values are positive and those where it is negative.

## CONTENT EXPECTATIONS FOR PRECALCULUS (CONT.)

- P4.2 Solve polynomial equations and inequalities of degree greater than or equal to three. Graph polynomial functions given in factored form using zeros and their multiplicities, testing the sign-on intervals and analyzing the function's large-scale behavior.
- P4.3 Know and apply fundamental facts about polynomials: the Remainder Theorem, the Factor Theorem, and the Fundamental Theorem of Algebra.

### P5 Rational Functions and Difference Quotients

- P5.1 Solve equations and inequalities involving rational functions. Graph rational functions given in factored form using zeros, identifying asymptotes, analyzing their behavior for large  $x$  values, and testing intervals.
- P5.2 Given vertical and horizontal asymptotes, find an expression for a rational function with these features.
- P5.3 Know and apply the definition and geometric interpretation of difference quotient. Simplify difference quotients and interpret difference quotients as rates of change and slopes of secant lines.

### P6 Trigonometric Functions

- P6.1 Define (using the unit circle), graph, and use all trigonometric functions of any angle. Convert between radian and degree measure. Calculate arc lengths in given circles.
- P6.2 Graph transformations of the sine and cosine functions (involving changes in amplitude, period, midline, and phase) and explain the relationship between constants in the formula and transformed graph.
- P6.3 Know basic properties of the inverse trigonometric functions  $\sin^{-1} x$ ,  $\cos^{-1} x$ ,  $\tan^{-1} x$ , including their domains and ranges. Recognize their graphs.
- P6.4 Know the basic trigonometric identities for sine, cosine, and tangent (e.g., the Pythagorean identities, sum and difference formulas, co-functions relationships, double-angle and half-angle formulas).

- P6.5** Solve trigonometric equations using basic identities and inverse trigonometric functions.
- P6.6** Prove trigonometric identities and derive some of the basic ones (e.g., double-angle formula from sum and difference formulas, half-angle formula from double-angle formula, etc.).
- P6.7** Find a sinusoidal function to model a given data set or situation and explain how the parameters of the model relate to the data set or situation.

## **P7 Vectors, Matrices, and Systems of Equations**

- P7.1** Perform operations (addition, subtraction, and multiplication by scalars) on vectors in the plane. Solve applied problems using vectors.
- P7.2** Know and apply the algebraic and geometric definitions of the dot product of vectors.
- P7.3** Know the definitions of matrix addition and multiplication. Add, subtract, and multiply matrices. Multiply a vector by a matrix.
- P7.4** Represent rotations of the plane as matrices and apply to find the equations of rotated conics.
- P7.5** Define the inverse of a matrix and compute the inverse of two-by-two and three-by-three matrices when they exist.
- P7.6** Explain the role of determinants in solving systems of linear equations using matrices and compute determinants of two-by-two and three-by-three matrices.
- P7.7** Write systems of two and three linear equations in matrix form. Solve such systems using Gaussian elimination or inverse matrices.
- P7.8** Represent and solve systems of inequalities in two variables and apply these methods in linear programming situations to solve problems.

## CONTENT EXPECTATIONS FOR PRECALCULUS (CONT.)

### **P8 Sequences, Series, and Mathematical Induction**

- P8.1** Know, explain, and use sigma and factorial notation.
- P8.2** Given an arithmetic, geometric, or recursively defined sequence, write an expression for the  $n$ th term when possible. Write a particular term of a sequence when given the  $n$ th term.
- P8.3** Understand, explain, and use the formulas for the sums of finite arithmetic and geometric sequences.
- P8.4** Compute the sums of infinite geometric series. Understand and apply the convergence criterion for geometric series.
- P8.5** Understand and explain the principle of mathematical induction and prove statements using mathematical induction.
- P8.6** Prove the binomial theorem using mathematical induction. Show its relationships to Pascal's triangle and to combinations. Use the binomial theorem to find terms in the expansion of a binomial to a power greater than 3.

### **P9 Polar Coordinates, Parameterizations, and Conic Sections**

- P9.1** Convert between polar and rectangular coordinates. Graph functions given in polar coordinates.
- P9.2** Write complex numbers in polar form. Know and use De Moivre's Theorem.
- P9.3** Evaluate parametric equations for given values of the parameter.
- P9.4** Convert between parametric and rectangular forms of equations.
- P9.5** Graph curves described by parametric equations and find parametric equations for a given graph.
- P9.6** Use parametric equations in applied contexts (e.g., orbits and projectiles) to model situations and solve problems.

- P9.7** Know, explain, and apply the locus definitions of parabolas, ellipses, and hyperbolas and recognize these conic sections in applied situations.
- P9.8** Identify parabolas, ellipses, and hyperbolas from equations, write the equations in standard form, and sketch an appropriate graph of the conic section.
- P9.9** Derive the equation for a conic section from given geometric information (e.g., find the equation of an ellipse given its two axes). Identify key characteristics (e.g. foci and asymptotes) of a conic section from its equation or graph.
- P9.10** Identify conic sections whose equations are in polar or parametric form.







**Michigan Department of Education**

Office of School Improvement

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